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Bianchi-I Space-time with Variable Gravitational and Cosmological "Constants"

C.P. Singh · Suresh Kumar

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Abstract We consider Einstein's field equations with variable gravitational and cosmological "constants" for a spatially homogeneous and anisotropic Bianchi-I space-time. A law of variation for the Hubble parameter, which is related to the average scale factor and yields a constant value of the deceleration parameter, is assumed to solve the field equations. The gravitational constant is allowed to follow a power-law form. We find that a time-increasing gravitational constant is suitable for describing the present evolution of universe. The solutions reveal the dynamics of a universe, which expands forever. The physical interpretation of the solutions is discussed in detail.

Keywords Bianchi space-time · Hubble parameter · Deceleration parameter · Gravitational constant · Cosmological constant

1 Introduction

One of the most important and outstanding problems in cosmology is the cosmological constant (Λ) problem (for excellent reviews see, Weinberg [1] and Padmanabhan [2]). Observations strongly favor a small and positive value of the effective cosmological constant at the present epoch [3–10]. Among the various solutions proposed for the cosmological constant problem, the phenomenologically simple one is of endowing the effective Λ with a variable dynamical degree of freedom, which allows it to relax to its present value in an expanding universe [11]. Berman [12] has favored the hypothesis $\Lambda \propto t^{-2}$ by adding an additional term to the usual energy-momentum tensor. Vishwakarma et al. [13] have studied axially symmetric Bianchi type-I models with perfect fluid distribution of matter and variable Λ, where the Λ-term represents the energy density of vacuum. In recent past, a number of

C.P. Singh · S. Kumar (✉)
Department of Applied Mathematics, Delhi College of Engineering, Bawana Road, Delhi 110 042, India
e-mail: ysuresh74@rediffmail.com

C.P. Singh
e-mail: cpsphd@rediffmail.com

authors have considered cosmological models with time-dependent cosmological constant (see, Singh and Kumar [14] and references therein).

The time variation of the gravitational constant G was first proposed by Dirac [15] in his large number hypothesis and since then it has been used frequently in numerous modifications of the general theory of relativity in order to meet various astronomical observations [16]. In recent years, an important modification has come into picture, which links the variation of G with that of the variable Λ-term within the framework of the general theory of relativity. An appealing feature of this modification is that it leaves the form of Einstein's equations formally unchanged by allowing the variation of G to be accompanied by a change in Λ and enables us to solve many cosmological problems such as the cosmological constant problem, inflationary scenario etc. [17]. This approach was first proposed by Lau [18] and since then many authors have investigated cosmological models with variable G and Λ (see, Singh et al. [19] and Oli [20] and references therein).

The astronomical observations have revealed that on large scale the universe is isotropic and homogeneous in its present state of evolution. But it might not be the same in the past. Therefore the models with anisotropic background that approach to isotropy at late times, are most suitable for describing the entire evolution of the universe. The spatially homogeneous and anisotropic Bianchi-I space-time provides such a framework. In the literature Bianchi type-I cosmological models with variable G and Λ have been studied by Singh and Agrawal [21], Beesham [22], Kalligas et al. [23], Arbab [24, 25], Beesham et al. [26], Kilinc [27], Vishwakarma [28], Chakraborty and Roy [29], Singh et al. [30–32], Tiwari [33] and Oli [20].

In this paper we study some physically realistic and anisotropic Bianchi type-I models with variable gravitational and cosmological constants. This work is organized as follows: The model and field equations are given in Sect. 2. The field equations are solved in Sect. 3 by assuming some physically relevant assumptions and the physical behavior of the solutions is discussed in detail. A special case of solutions is discussed in Sect. 4. The concluding remarks are summarized in Sect. 5.

2 Model and Field Equations

The spatially homogeneous and anisotropic Bianchi-I space-time is described by the line element

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2, \tag{1}$$

where $A(t)$, $B(t)$ and $C(t)$ are the metric functions of cosmic time t.

We define $a = (ABC)^{\frac{1}{3}}$ as the average scale factor so that the generalized Hubble parameter in anisotropic models may be defined as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \tag{2}$$

where an over dot denotes derivative with respect to the cosmic time t.

The directional Hubble parameters along x, y and z coordinate axes, respectively, may be defined as

$$H_1 = \frac{\dot{A}}{A}, \qquad H_2 = \frac{\dot{B}}{B}, \qquad H_3 = \frac{\dot{C}}{C}. \tag{3}$$

The Einstein's field equations with time-dependent G and Λ are given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -8\pi G(t)T_{ij} + \Lambda(t)g_{ij}, \tag{4}$$

where T_{ij} is stress energy tensor of matter which, in case of perfect fluid, has the form

$$T_{ij} = (\rho + p)u_i u_j + pg_{ij}, \tag{5}$$

where ρ is the matter density; p is thermodynamic pressure and u^i is four-velocity vector satisfying $u^i u_i = -1$.

In the field equations (4), Λ accounts for vacuum energy with its energy density ρ_{v} and pressure p_{v} satisfying the equation of state

$$p_{\mathrm{v}} = -\rho_{\mathrm{v}} = -\frac{\Lambda}{8\pi G}. \tag{6}$$

The critical density and the density parameters for energy density and cosmological constant are, respectively, defined as

$$\rho_{\mathrm{c}} = \frac{3H^2}{8\pi G}, \tag{7}$$

$$\Omega_{\mathrm{M}} = \frac{\rho}{\rho_{\mathrm{c}}} = \frac{8\pi G\rho}{3H^2}, \tag{8}$$

$$\Omega_{\Lambda} = \frac{\rho_{\mathrm{v}}}{\rho_{\mathrm{c}}} = \frac{\Lambda}{3H^2}. \tag{9}$$

We observe that the density parameters Ω_{M} and Ω_{Λ} are singular when $H = 0$.

In a co-moving coordinate system, the field equations (4), for the anisotropic Bianchi type-I space-time (1), in case of (5), read as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi Gp + \Lambda, \tag{10}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -8\pi Gp + \Lambda, \tag{11}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi Gp + \Lambda, \tag{12}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = 8\pi G\rho + \Lambda. \tag{13}$$

The covariant divergence of (4) yields

$$\dot{\rho} + 3(\rho + p)H + \rho\frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G} = 0. \tag{14}$$

The usual energy conservation equation $T^{ij}_{;j} = 0$, leads to

$$\dot{\rho} + 3(\rho + p)H = 0. \tag{15}$$

Now (14) reduces to

$$\rho\frac{\dot{G}}{G}+\frac{\dot{\Lambda}}{8\pi G}=0. \tag{16}$$

The field equations (10)–(13) are observed to involve seven unknown variables viz. A, B, C, ρ, p, G and Λ. Therefore to solve the field equations explicitly together with the energy conservation relation (15), we need two extra relations among the unknown variables, which we shall consider in the following section and solve the field equations exactly.

3 Solution of Field Equations

Subtracting (10) from (11), (10) from (12), (11) from (12) and taking second integral of each, we get the following three relations respectively:

$$\frac{A}{B}=d_1\exp\left(x_1\int a^{-3}dt\right), \tag{17}$$

$$\frac{A}{C}=d_2\exp\left(x_2\int a^{-3}dt\right), \tag{18}$$

$$\frac{B}{C}=d_3\exp\left(x_3\int a^{-3}dt\right), \tag{19}$$

where d_1, x_1, d_2, x_2, d_3 and x_3 are constants of integration.

From (17)–(19), the metric functions can be explicitly written as

$$A(t)=a_1 a\exp\left(b_1\int a^{-3}dt\right), \tag{20}$$

$$B(t)=a_2 a\exp\left(b_2\int a^{-3}dt\right), \tag{21}$$

$$C(t)=a_3 a\exp\left(b_3\int a^{-3}dt\right), \tag{22}$$

where

$$a_1=\sqrt[3]{d_1d_2},\qquad a_2=\sqrt[3]{d_1^{-1}d_3},\qquad a_3=\sqrt[3]{(d_2d_3)^{-1}},$$

$$b_1=\frac{x_1+x_2}{3},\qquad b_2=\frac{x_3-x_1}{3},\qquad b_3=\frac{-(x_2+x_3)}{3}.$$

It deserves to mention that these constants satisfy the following two relations:

$$a_1a_2a_3=1,\qquad b_1+b_2+b_3=0. \tag{23}$$

From (20)–(22), we observe that an explicit form of the average scale factor a would be helpful in determining the scale factors. In order to achieve the same we consider that the generalized Hubble parameter H is related to the average scale factor a by the relation

$$H=Da^{-n}, \tag{24}$$

where D and n are positive constants. This type of relation yields a constant value of deceleration parameter (DP) and has already been considered by Berman [34] and Berman and Gomide [35] for solving Friedmann-Robertson-Walker (FRW) models. They have shown that the different phases of the evolution of universe viz. radiation, inflation and pressure-free phases, may be considered as the particular cases of the $q =$ constant.

The DP q, an important observational quantity, is defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{25}$$

From (2) and (24), we get

$$\dot{a} = Da^{-n+1}, \tag{26}$$

$$\ddot{a} = -D^2(n-1)\,a^{-2n+1}. \tag{27}$$

Integration of (27) gives

$$a = (nDt)^{\frac{1}{n}}, \tag{28}$$

where the constant of integration has been omitted by assuming that $a = 0$ at $t = 0$. Substituting (26)–(28) into (25), we get

$$q = n - 1. \tag{29}$$

This shows that the value of DP is constant. It is interesting to note that the works of Perlmutter et al. [5–7], Riess et al. [8, 9], Tonry et al. [36], Knop et al. [37] and John [38], on the basis of the recent observations of type Ia supernovae, reveal the approximate value of DP in the range $-1 < q < 0$, which can be obtained from the relation (29) by restricting n in the range $0 < n < 1$. Thus the relevance of the relation (24) is justified. In a series of works, Singh and Kumar [14, 39–42] and Kumar and Singh [43–45] have also utilized the above form of Hubble parameter for obtaining exact solutions of anisotropic Bianchi type-I and II cosmological models in general relativity and some scalar-tensor theories of gravitation.

We further assume a power-law form of the gravitational constant (G) as [46]

$$G \propto t^m, \tag{30}$$

where m is a constant. For the sake of mathematical simplicity, (30) may again be written as

$$G = G_0(nDt)^m, \tag{31}$$

where G_0 is a positive constant.

Now, using (28) into (20)–(22), we get the following expressions for scale factors:

$$A(t) = a_1(nDt)^{\frac{1}{n}} \exp\left[\frac{b_1}{D(n-3)}(nDt)^{\frac{n-3}{n}}\right], \tag{32}$$

$$B(t) = a_2(nDt)^{\frac{1}{n}} \exp\left[\frac{b_2}{D(n-3)}(nDt)^{\frac{n-3}{n}}\right], \tag{33}$$

$$C(t) = a_3(nDt)^{\frac{1}{n}} \exp\left[\frac{b_3}{D(n-3)}(nDt)^{\frac{n-3}{n}}\right], \tag{34}$$

where $n \neq 3$. We shall discuss the cosmology for the special case $n = 3$ in the next section.

Substituting (32)–(34) into (12) and (13), we get

$$8\pi Gp - \Lambda = D^2(2n-3)(nDt)^{-2} - \beta(nDt)^{\frac{-6}{n}}, \tag{35}$$

$$8\pi G\rho + \Lambda = 3D^2(nDt)^{-2} - \beta(nDt)^{\frac{-6}{n}}, \tag{36}$$

where $\beta = b_1^2 + b_2^2 + b_1 b_2$.

Using (31) into (35) and (36), and then solving the resulting equations with (15), we get the following set of solutions:

$$\rho = \frac{3}{4\pi G_0}\left[\frac{D^2}{m+2}(nDt)^{-(m+2)} - \frac{\beta}{mn+6}(nDt)^{\frac{-(mn+6)}{n}}\right], \tag{37}$$

$$p = \frac{1}{4\pi G_0}\left[\frac{(mn+2n-3)D^2}{m+2}(nDt)^{-(m+2)} - \frac{(mn+3)\beta}{mn+6}(nDt)^{\frac{-(mn+6)}{n}}\right], \tag{38}$$

$$\Lambda = \frac{3mD^2}{m+2}(nDt)^{-2} - \frac{mn\beta}{mn+6}(nDt)^{\frac{-6}{n}}, \tag{39}$$

provided $m \neq -2, \frac{-6}{n}$. The constant of integration appeared in the process of calculation has been taken as zero for simplicity.

We find that the above solutions satisfy (16) identically and hence represent exact solutions of the Einstein's field equations (10)–(13). It is interesting to observe that if we assume $a_1 = a_2 = a_3$, $\beta = 0$ and some particular values of the parameters m and n, the above solutions exactly reduce to the isotropic models with variable G and Λ investigated by Kalligas et al. [46] and Berman [47].

The physical parameters such as directional Hubble parameters (H_i), Hubble parameter (H), expansion scalar (θ), spatial volume (V), anisotropy parameter ($\bar{A}$) and shear scalar (σ^2) are, respectively, given by

$$H_i = (nt)^{-1} + b_i(nDt)^{\frac{-3}{n}} \quad (i = 1, 2, 3), \tag{40}$$

$$H = (nt)^{-1}, \tag{41}$$

$$\theta = u^i_{;i} = \frac{3\dot{a}}{a} = 3(nt)^{-1}, \tag{42}$$

$$V = (nDt)^{\frac{3}{n}}, \tag{43}$$

$$\bar{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{1}{3D^2}(b_1^2 + b_2^2 + b_3^2)(nDt)^{\frac{2n-6}{n}}, \tag{44}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \beta(nDt)^{\frac{-6}{n}}. \tag{45}$$

The vacuum energy density (ρ_v), critical density (ρ_c) and the density parameters (Ω_M, Ω_Λ) read as

$$\rho_v = \frac{1}{8\pi G_0}\left[\frac{3mD^2}{m+2}(nDt)^{-(m+2)} - \frac{mn\beta}{mn+6}(nDt)^{\frac{-(mn+6)}{n}}\right], \tag{46}$$

$$\rho_c = \frac{3D^2}{8\pi G_0}(nDt)^{-(m+2)}, \tag{47}$$

$$\Omega_{\rm M} = \frac{2}{m+2} - \frac{2\beta}{D^2(mn+6)}(nDt)^{\frac{2n-6}{n}}, \tag{48}$$

$$\Omega_{\Lambda} = \frac{m}{m+2} - \frac{mn\beta}{3D^2(mn+6)}(nDt)^{\frac{2n-6}{n}}. \tag{49}$$

Adding (48) and (49), we get

$$\Omega_{\rm total} = \Omega_{\rm M} + \Omega_{\Lambda} = 1 - \frac{\beta}{3D^2}(nDt)^{\frac{2n-6}{n}}. \tag{50}$$

For $\beta = 0$, we have

$$\begin{aligned} &H_i = H = (nt)^{-1}, \qquad \sigma^2 = 0, \\ &\Omega_{\rm M} = \frac{2}{m+2}, \quad \Omega_{\Lambda} = \frac{m}{m+2} \quad \text{and} \quad \Omega_{\rm total} = 1. \end{aligned} \tag{51}$$

This shows that the density parameters depend on the values of m. Moreover the resulting model would have $\rho = \rho_c$.

For $\beta \neq 0$, (50) yields

$$\frac{\rho_{\rm total}}{\rho_c} = 1 - \frac{\beta}{3D^2}(nDt)^{\frac{2n-6}{n}}, \tag{52}$$

where $\rho_{\rm total} = \rho + \rho_v$ is the overall energy density of the universe. Equation (52) reveals that the net energy density ($\rho_{\rm total}$) stays less than the critical density (ρ_c) all the time. It follows that the universe will never collapse and will continue to expand forever.

For $m = 0$ i.e. $G = \text{const.}$, the vacuum energy density (ρ_v) vanishes, which in turn implies that creation of matter does not take place in the model. The physical requirement that the energy density ρ must decrease with time, is fulfilled provided $m > -2$. If $m < 0$, the gravitational constant G decreases with time. The same continues to increase with time as long as $m > 0$. It follows that physically realistic cosmological models are possible with increasing as well as decreasing gravitational constant with time.

It is observed that the spatial volume is zero at $t = 0$ and expansion scalar is infinite, which shows that the universe starts evolving with zero volume at $t = 0$ with a big bang. The scale factors also vanish at $t = 0$ and hence the model has a point singularity at the initial epoch. The pressure, energy density, cosmological constant, Hubble parameter, expansion scalar, shear scalar, vacuum energy density and critical density diverge at the initial singularity. The anisotropy parameter and density parameters diverge at the initial epoch provided $n < 3$. The gravitational constant also diverges provided $m < 0$ otherwise it is a non-negative constant at the initial singularity.

As $t \to \infty$, the scale factors and volume become infinite whereas ρ, p, Λ, H_1, H_2, H_3, θ, $\bar{A}$, σ^2, ρ_v and ρ_c tend to zero. The ratio σ/θ, which is very large initially, tends to zero as $t \to \infty$ provided $n < 3$. So the model approaches isotropy for large values of t. Thus the model represents shearing, non-rotating and expanding model of the universe with a big bang start approaching to isotropy at late times.

Now we analyze the variation of various parameters graphically in cosmological models with time-decreasing, constant and time-increasing gravitational constant G. We consider the cases $m = -1$, $m = 0$ and $m = 1$, and name the corresponding models as G-decreasing, G-constant and G-increasing universes respectively.

Figure 1 shows that the energy density falls gradually as we move from $m = -1$ to $m = 1$. In other words the G-decreasing universe has higher energy density all the time than

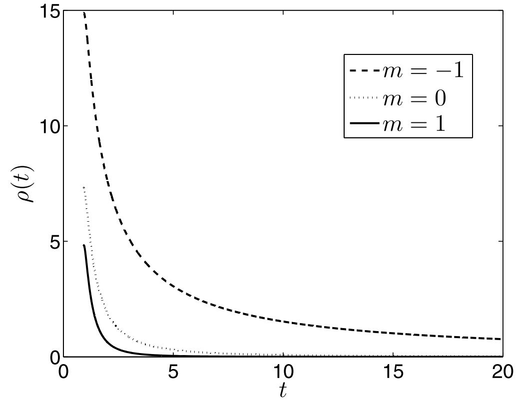

Fig. 1 Plot of $\rho(t)$ versus t with $n = 0.5$, $D = 2$ and $\beta = 1$

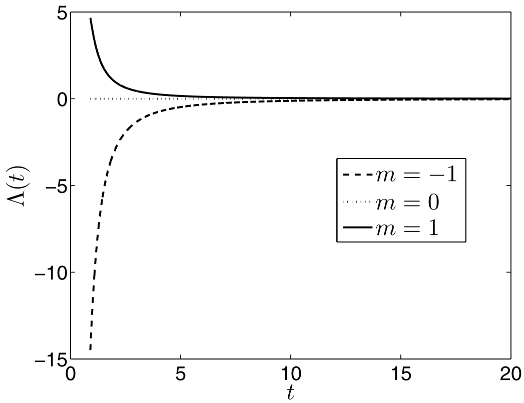

Fig. 2 Plot of $\Lambda(t)$ versus t with $n = 0.5$, $D = 2$, $G_0 = 1$ and $\beta = 1$

the G-increasing universe, whereas the energy density of the G-constant universe stays in between the two. However in each case the energy density decreases to zero at late times.

Figure 2 reveals that the cosmological constant Λ remains negative in a G-decreasing universe. However the present observations support a positive value of cosmological constant. It immediately follows that the form of variable G viz. $G \propto t^{-1}$ proposed by Dirac [15] in his large number hypothesis, is not suitable to meet the present astronomical observations.

Further the dotted horizontal straight line in Fig. 2 indicates that the cosmological constant remains zero during the entire evolution of a G-constant universe. In the G-increasing universe, the cosmological constant stays positive all the time. However it decreases as the evolution of the universe progresses and at late times it becomes negligible. This is in accordance with the recent cosmological observations [3–10], which support a small and positive value of the cosmological constant Λ. Thus we conclude that the G-increasing models can be helpful in resolving the so-called cosmological constant problem and hence are suitable for describing the dynamics of the universe at the present epoch.

From Fig. 3, we observe that negative pressure dominates the universe in each case, where $n = 0.5$ i.e. $q = -0.5$. Therefore negative pressure may be responsible for the accelerated expansion of the universe.

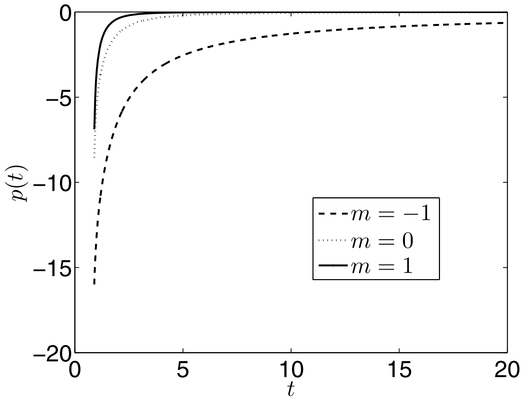

Fig. 3 Plot of $p(t)$ versus t with $n=0.5$, $D=2$, $G_0=1$ and $\beta=1$

4 Cosmology for the Special Case $n=3$

In this case, the average scale factor and the metric functions read as

$$a(t)=(3Dt)^{\frac{1}{3}}, \tag{53}$$

$$A(t)=a_1(3Dt)^{\frac{D+b_1}{3D}}, \tag{54}$$

$$B(t)=a_2(3Dt)^{\frac{D+b_2}{3D}}, \tag{55}$$

$$C(t)=a_3(3Dt)^{\frac{D+b_3}{3D}}. \tag{56}$$

The other unknown variables of the field equations have the following expressions:

$$G=G_0(3Dt)^m, \tag{57}$$

$$\rho=\frac{3D^2-\beta}{4\pi G_0(m+2)}(3Dt)^{-(m+2)}, \tag{58}$$

$$p=\frac{(3D^2-\beta)(m+1)}{4\pi G_0(m+2)}(3Dt)^{-(m+2)}, \tag{59}$$

$$\Lambda=\frac{(3D^2-\beta)m}{m+2}(3Dt)^{-2}. \tag{60}$$

The cosmological parameters of physical importance for the model are given by

$$H_i=(3D+b_i)(3Dt)^{-1}\quad (i=1,2,3), \tag{61}$$

$$H=\frac{1}{3}t^{-1}, \tag{62}$$

$$\theta=t^{-1}, \tag{63}$$

$$V=3Dt, \tag{64}$$

$$\bar{A}=\frac{1}{3D^2}(b_1^2+b_2^2+b_3^2), \tag{65}$$

$$\sigma^2=\beta(nDt)^{-2}, \tag{66}$$

$$\rho_{\mathrm{v}} = \frac{(3D^2-\beta)m}{4\pi G_0(m+2)}(3Dt)^{-(m+2)}, \tag{67}$$

$$\rho_{\mathrm{c}} = \frac{3D^2}{8\pi G_0}(3Dt)^{-(m+2)}, \tag{68}$$

$$\Omega_M = \frac{2(3D^2-\beta)}{3D^2(m+2)}, \tag{69}$$

$$\Omega_\Lambda = \frac{m(3D^2-\beta)}{3D^2(m+2)}, \tag{70}$$

$$\Omega_{\mathrm{total}} = 1 - \frac{\beta}{3D^2}. \tag{71}$$

We observe that for the existence and physical validity of the above solutions, the suitable conditions are $m > -2$ and $3D^2 > \beta$. Further we find that the above solutions satisfy (16) identically and exactly reduce to the isotropic solutions investigated by Lau [18] and Beesham [48] if we choose $\beta = 0$, $a_1 = a_2 = a_3$ and $m = -1$. One can also obtain the solutions for Bianchi-I space-time with variable G and Λ, recently presented by Singh et al. [30, 31], by choosing some particular values of b_1, b_2, b_3 and m in the above solutions.

It is observed that the model starts with a bag bang at $t = 0$ and evolves with a power-law expansion. As the evolution progresses, the volume of the universe increases linearly whereas energy density, pressure, cosmological constant, Hubble parameter, expansion scalar, vacuum energy density and critical density decrease with the cosmic time t. The cosmological constant vanishes for $m = 0$, which leads to a constant value for gravitational constant. For $m \neq 0$, the cosmological constant remains positive during the evolution. The anisotropy parameter has a time-independent value and therefore the model remains anisotropic during the entire evolution. The density parameters also remain constants during the evolution in such a way that overall energy density stays less than the critical density. Therefore the model expands forever.

5 Conclusion

In this paper we have studied a spatially homogeneous and anisotropic Bianchi-I space-time with time-varying G and Λ. The field equations have been solved exactly by using a special variation of generalized Hubble parameter that yields a constant value of DP. Exact and physically viable Bianchi type-I models have been obtained, which generalize the works of several authors as discussed in Sects. 3 and 4. Therefore the approach used in this paper to find the exact solutions, is more compact than others and has provided more general and new solutions for the Bianchi-I space-time with variable G and Λ. The models have singular origin and are found to be highly anisotropic in the early stages of evolution. The anisotropy goes off during the process of evolution except in the case $n = 3$ and we get the standard isotropic model at late times.

For graphical analysis of the solutions, we have considered three different cases viz. $G \propto t^{-1}$, $G = \text{const.}$ and $G \propto t$. The time-increasing gravitational constant has been found to favor the present evolution of universe. The cosmological constant remains positive and decays with the cosmic evolution. Therefore it is understandable why the observations favor a small and positive value of the cosmological constant in the present universe. The model presented for $n = 3$, retains constant anisotropy during the entire evolution. The energy density of the whole content of the universe in the models has been found to be less than the critical density during the entire cosmic evolution and thereby setting the universe to expand forever. For $m = 0$, we get $G = \text{const.}$, $\Lambda = 0$ and the solutions presented in our earlier work [43] in the standard general relativity. Thus we have extended our earlier work [43] by obtaining exact solutions in the modified general relativity, where G and Λ vary with time. Finally, the solutions presented in this paper are new and may be useful for better understanding of the evolution of universe in Bianchi-I space-time with variable gravitational and cosmological constants.

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